

## Even numbers

Let  $m$  and  $n$  be positive integers

a)  $2n$  and  $2m$  are even numbers.

Why?

b)  $2n$  and  $2m$  are the same even number if  $n = m$

Why?

c) Show that the product of two distinct even numbers is also even.

d) Show that the square of an even number is also even.

e) Show that the sum of two even numbers is even.

## Odd numbers

Let  $m$  and  $n$  be positive integers

a) As  $m$  and  $n$  are positive integers,  $2m$  and  $2n$  are both even  $2n - 1$  and  $2m - 1$  are odd numbers. Why?

b) Show that the product of two distinct odd numbers is also odd.

c) Show that the square of an odd number is also odd.

## Mathematical arguments

a) Show algebraically that

$$4(3x - 5) - 3 \equiv 12x - 23$$

b) Show algebraically that

$$2(n + 2)(n - 1) - 2(1 - n) \equiv (n - 1)(n + 3)$$

c) Show algebraically that

$$(n + 7)^2 - (n + 1)^2 \equiv 12(n + 4)$$

## Consecutive numbers

$n$ ,  $n + 1$  and  $n + 2$  are consecutive numbers.

How would you write three consecutive even numbers?

## Giving counterexamples

a) The sequence generated by  $4n - 1$  contains infinitely many primes, but not every number in this sequence is prime. Give an example of a number from this sequence that is not prime.

b) Shaq thinks that  $(3n)^3$  is always positive if  $n$  is an integer. Find a counterexample to show that Shaq is incorrect.